

Estimating \hat{q} in Quenched Lattice SU(2) Gauge Theory

Abhijit Majumder

Department of Physics and Astronomy, Wayne State University, Detroit, Michigan, 48201, USA.

Abstract

The propagation of a virtual quark in a thermal medium is considered. The non-perturbative jet transport coefficient \hat{q} is estimated in quark less SU(2) lattice gauge theory. The light like correlator which defines \hat{q} , defined in the regime where the jet has small virtuality compared to its energy, is analytically related to a series of local operators in the deep Euclidean region, where the jet's virtuality is of the same order as its energy. It is demonstrated that in this region, for temperatures in the range of $T=400-600$ MeV, and for jet energies above 20 GeV, the leading term in the series is dominant over the next-to-leading term and thus yields an estimate of the value of \hat{q} . In these proceedings we discuss the details of the numerical calculation.

1. Introduction

In several formulations of jet modification [1], transport coefficients play a crucial role. These, in principle, non perturbative quantities encode the effect of the medium on the modification encountered by the hard jet. In the Higher-Twist (HT) approach [2], transport coefficients represent the expectation values of operator products that have been factorized from the hard process of jet scattering and radiative emission in the medium. In the standard HT formalism, these are considered entirely non-perturbative and no attempt is made to evaluate them. In these proceedings, we outline the results of a reformulation of the HT scheme allowing for the evaluation of these coefficients in lattice gauge theory at finite temperature. The modification of the formalism has been described in Refs. [3]. Here we discuss the pertinent numerical results.

2. Formulation in Minkowski Space-Time

Consider a single quark with a large negative light-cone momentum q^- propagating through a box of length L , held at a fixed temperature $T = 1/\beta$, undergoing a single scattering which imparts transverse momentum to the quark. The quark then exits the medium and is observed. The length of the medium will be considered to be short enough such that the quark does not radiate during traversal. The coupling of the jet with the medium will be considered to be small enough such that secondary scattering is minimal. Dividing the square of the transverse momentum gained by L , we obtain the non-perturbative definition of the transport coefficient \hat{q} , i.e.,

$$\hat{q} = \frac{4\pi^2\alpha_s}{N_c} \int \frac{dy^- d^2y_\perp}{(2\pi)^3} d^2k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp \cdot \vec{y}_\perp} \langle n | \frac{e^{-\beta E_n}}{Z} F^{+, \perp}(y^-, y_\perp) F_\perp^+(0) | n \rangle. \quad (1)$$

In the equation above, y^- is a light-cone separation, $F^{\mu\nu} = t^a F^{a\mu\nu}$ is the gauge field strength, $|n\rangle$ is a state of the thermal ensemble with a partition function Z , α_s is the strong coupling constant, and N_c is the dimensionality of the fundamental representation.

Preprint submitted to Nuclear Physics A

March 5, 2013

Compared to the large energy of the hard parton, $q^- \sim Q$ (where Q denotes the hardest scale in the problem), the transverse momentum imparted to the parton (k_\perp) is considered to be rather small, $k_\perp \sim \lambda Q$, where $\lambda \ll 1$ is a small dimensionless constant. As a result, the (+)-component of the the momentum transferred, $k^+ \sim k_\perp^2/(2q^-)$ is of the order of $\lambda^2 Q$. As argued in Ref. [3], this operator product, \hat{q} , defined over values of $k^+ = k_\perp^2/(2q^-)$ may be expressed as the imaginary part of the more generalized operator product,

$$\hat{Q} = \frac{4\pi^2\alpha_s}{N_c} \int \frac{d^4y d^4k}{(2\pi)^4} e^{ik \cdot y} \frac{2(q^-)^2}{\sqrt{2}q^-} \frac{\langle M|F^{+\perp}(0)F_{\perp}^+(y)|M\rangle}{(q+k)^2 + i\epsilon}. \quad (2)$$

As defined above, \hat{Q} has a branch cut in the regime where $q^+ \sim \lambda^2 Q$, the discontinuity across the cut yields \hat{q} . One may then define the following integral in the deep Euclidean region,

$$I_1 = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)}, \quad (3)$$

where, the contour is taken around the point $q^+ = -q^-$. We now evaluate the integral using the method of residues, at $q^+ = -q^-$. Since q^- is very large compared to k , one may expand out the denominator, and the y^- dependence in the numerator to obtain,

$$I_1 = \frac{2\sqrt{2}\pi^2\alpha_s}{N_c q^-} \langle M|F_\perp^{+\mu} \sum_{n=0}^{\infty} \left(\frac{-i\mathcal{D}^0}{q^-} \right)^n F_{\perp,\mu}^+ |M\rangle. \quad (4)$$

The above equation represents a series of local operators, containing rising powers of derivatives in the numerator along with similarly rising powers of the hard scale q^- in the denominator. In any evaluation of I_1 , the presence of a large energy scale q^- will allow one to terminate the series at a reasonably small value of n while encumbering a minimal amount of error.

We can now deform the contour and evaluate it over the branch cut from $q^+ > -\lambda^2 Q$ to $q^+ \rightarrow \infty$. This yields,

$$\begin{aligned} I_1 &= \frac{4\pi^2\alpha_s}{N_c} \int dq^+ \frac{d^4y d^4k}{(2\pi)^4} e^{ik \cdot y} \frac{\delta\left(k^+ + q^+ - \frac{k_\perp^2}{2q^-}\right)}{2q^-} \frac{\langle M|F^{+\mu}(0)F_{\mu}^+(y)|M\rangle}{(q^+ + q^-)} \\ &= \int_{-\lambda^2 Q}^{\lambda^2 Q} dq^+ \frac{\hat{q}(q^+)}{q^+ + q^-} + \int_0^\infty dq^+ V(q^+). \end{aligned} \quad (5)$$

The second term in the equation above, refers to the contribution to the operator above from vacuum gluon radiation, i.e., the Bremsstrahlung radiation of gluons from an off-shell quark. As such, it contributes only in the region where the virtuality of the incoming quark is time-like and is independent of the temperature of the medium. Thus when T is varied, the second term above is a constant, while the first depends on the temperature of the medium. Comparing the two equations (4) and (5), subtracting the pure vacuum contributions, one may evaluate \hat{q} . In the remainder, we discuss the the evaluation of the operators in Eq. (4).

3. Euclidean space and results of calculations

All the operator products in Eq. (4) represent local operators which may be easily evaluated by rotation to Euclidean space. Denoting a generic Minkowski space operator product as $\mathcal{D}^>(t) =$

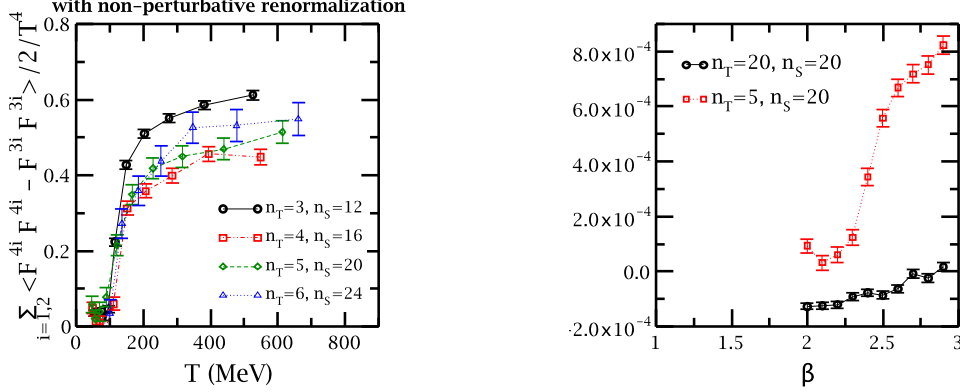


Figure 1: Left panel: The temperature dependence of the local operator $\langle F^{+i} F^{+i} \rangle$ scaled by T^4 to make it dimensionless. The lattice spacing is set using a non-perturbative RG prescription. Right panel: Unscaled expectation of lattice-size-independent correlator $\sum_{i=1,2} a_L^4 (F^{3i} F^{3i} - F^{4i} F^{4i})/2$ at finite temperature (red squares), versus expectation in vacuum (black circles), as a function of $\beta = 4/g^2$ (g is the bare lattice coupling). The plot is for $n_t = 5, n_s = 20$.

$\sum_n \langle n | e^{-\beta H} O_1(t) O_2(0) | n \rangle$, we obtain the simple relation that, $\mathcal{D}^>(t=0) = i^{N_t} \Delta(\tau=0)$. Where, N_t denotes the number of time derivatives in $\mathcal{D}^>(t)$. Using the above relation, local operator products in Minkowski space may be obtained from local operator products in Euclidean space.

We report results on a $(4 \times n_t)^3 \times n_t$ lattice where n_t is varied from 3 to 6. All calculations have been done with 5000 heat bath sweeps for each data point. We have used the Wilson gauge action for SU(2) [4, 5]. The scale is set on the lattice using two different renormalization group (RG) formulas: The first is based on the two loop perturbative RG equation for the string tension [4, 5], which yields the following formula for the lattice spacing,

$$a_L = \frac{1}{\Lambda_L} \left(\frac{11g^2}{24\pi^2} \right)^{-\frac{51}{121}} \exp \left(-\frac{12\pi^2}{11g^2} \right), \quad (6)$$

where, g represents the bare lattice coupling and Λ_L represents the one dimension-full parameter on the lattice. Comparing with the vacuum string tension, we have used $\Lambda_L = 5.3$ MeV. For a lattice at finite temperature or one with $n_t \ll n_s$, the temperature is obtained as $T = 1/(n_t a_L)$. The results for the field-strength-field-strength correlation (FFC), $\sum_{i=1,2} (F^{3i} F^{3i} - F^{4i} F^{4i})/2$ with this choice of formula for the lattice spacing are presented in the left panel of Fig 2. The resulting correlation is scaled by T^4 , with T as obtained from the formula above.

We have also set the scale using a non-perturbative approach as outlined in Ref. [6], where the formula for the lattice spacing is expressed as the product of that obtained from Eq. (6) and a non-perturbative function $\lambda(g^2)$ which has been dialed to ensure that T_c/Λ_L is independent of g^2 . The results for the FFC, $\sum_{i=1,2} (F^{3i} F^{3i} - F^{4i} F^{4i})/2$ with this next choice of formula for the lattice spacing are presented in the left panel of Fig 1. While the scaling with lattice size is seen to be much better with the non-perturbative RG equation, for temperatures above $T = 400$ MeV, both calculations yield the same value of the leading operator product in the series for I_1 . In the following we will only focus on the results above a $T = 400$ MeV. As a result, we will use the perturbative RG equation to set the scale in the evaluation of the next-to-leading terms. The field-strength-field-strength correlation, plotted in the left panels of Fig. 1 and Fig. 2 contain

their vacuum parts, these have not been subtracted out. In the right panel of Fig. 1, we plot the unscaled correlator as a function of the coupling on the lattice and also plot the vacuum piece. For the points above the transition, the vacuum contribution is seen to be a small correction, and thus was ignored in all other plots.

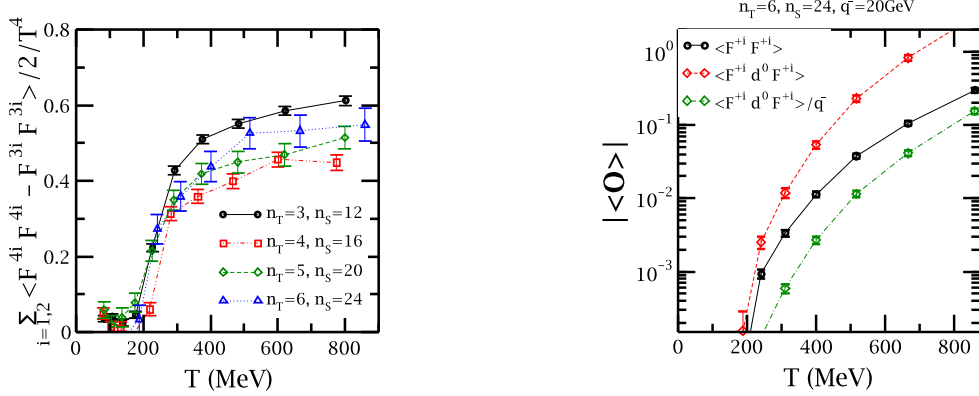


Figure 2: Left panel: The temperature dependence of the local operator $\langle F^{+i} F^{+i} \rangle$ scaled by T^4 to make it dimensionless. Right panel: temperature dependence of absolute values of the local operator $\langle F^{+i} F^{+i} \rangle$ and the next-to-leading operator products $\langle F^{+i} i \mathcal{D}^4 F^{+i} \rangle$ [red diamond (dashed)] and $\langle F^{+i} i \mathcal{D}^4 F^{+i} \rangle / q^-$ [green diamond (dot dashed)].

The next-to-leading operator product in the series for I_1 is plotted in the right panel of Fig. 2. Note, this operator has an extra temporal derivative and suppressed by a factor of q^- . The results of the lattice calculation of this operator are plotted, both with and without this large factor in the denominator (q^- is chosen to be 20 GeV). For temperatures below $T = 600$ MeV, this next-to-leading correction is found to be less than 20% of the leading term. As a result, in the range of temperatures from 400-600 MeV, the first term in the series expansion of I_1 can be used to obtain an estimate for \hat{q} . The phenomenology of these estimates is presented in Ref. [3]; for the case of a SU(2) quark traversing a SU(2) plasma, we obtain a virtuality averaged $\hat{q} = 0.186 \text{ GeV}^2/\text{fm}$, at a $T = 400$ MeV. Naively scaling this result with the number of colors and flavors for the case of an SU(3) gluon traversing an SU(3) plasma with two light flavors of quarks, we obtain $\hat{q} = 1.3 - 3.3 \text{ GeV}^2/\text{fm}$ at a $T = 400$ MeV.

This work was supported by the NSF under grant no PHY-1207918, and partially by the U.S. Department of Energy under grant no. de-sc0004286 and (within the framework of the JET collaboration) de-sc0004104. Computations were carried out on the Wayne State Grid.

References

- [1] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B **484**, 265 (1997); A. Majumder, Phys. Rev. D **85**, 014023 (2012); G. -Y. Qin, J. Ruppert, S. Turbide, C. Gale, C. Nonaka and S. A. Bass, Phys. Rev. C **76**, 064907 (2007); S. A. Bass, C. Gale, A. Majumder, C. Nonaka, G. -Y. Qin, T. Renk and J. Ruppert, Phys. Rev. C **79**, 024901 (2009).
- [2] X. -N. Wang and X. -f. Guo, Nucl. Phys. A **696**, 788 (2001) [hep-ph/0102230].
- [3] A. Majumder, arXiv:1202.5295 [nucl-th]; A. Majumder, arXiv:1210.3352 [nucl-th].
- [4] J. Engels, F. Karsch, H. Satz and I. Montvay, Phys. Lett. B **101**, 89 (1981).
- [5] M. Creutz, Cambridge, UK: Univ. Pr. (1983) 169 P. (Cambridge Monographs On Mathematical Physics).
- [6] J. Engels, F. Karsch and K. Redlich, Nucl. Phys. B **435**, 295 (1995).